Review (Mid Term #2)

Ex. Let $f(x) = \frac{1}{1-4x}$. Find the equation of tangent at $x = 0$.

- $f(x) = \frac{1}{1-4x} \Rightarrow f(0) = \frac{1}{1-4(0)} = 1.$

  So, $(0,1)$ point is on the graph of $f(x)$.

We are seeking the tangent of $f(x)$ at $(0,1)$ point.

- Slope of $f(x)$ at $x=0$ is:

$$f'(x) = -\frac{1}{(1-4x)^2} \cdot (-4) = \frac{4}{(1-4x)^2}$$

$$f'(0) = \frac{4}{(1-4(0))^2} = \frac{4}{1} = 4.$$

So the equation of the tangent passing through $(0,1)$ with slope at $x=0$ is 4 is given by:

$$y - 1 = 4(x-0)$$

$$\Rightarrow y = 1 + 4x.$$

Ex. Find the linear appoximation for $\frac{1}{1-4\left(\frac{1}{e^5}\right)}$, $\frac{1}{e^5} = 0.0067$

By above: $L(x) = 1 + 4x$

So, $f\left(\frac{1}{e^5}\right) \approx 1 + 4\left(\frac{1}{e^5}\right) = 1 + 4(0.0067)$
$= 1.0268$

Actually: $f\left(\frac{1}{e^5}\right) = \frac{1}{1-4\left(\frac{1}{e^5}\right)} = 1.02769$

Note: $\cdot\ \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$, $\frac{d}{dx}[\ln x] = \frac{1}{x}$

- $\log_t(ab) = \log_t a + \log_t b$
- $\log_t\left(\frac{a}{b}\right) = \log_t(a) - \log_t(b)$
- $\log_t(a^r) = r \log_t a$.

Ex. Find $\frac{d}{dx}\left[\log_2\left(\frac{x^2(3x+7)}{\sqrt{x^5+2}}\right)\right]$

Best Method: Use log properties first.

$$\log_2\left(\frac{x^2(3x+7)}{\sqrt{x^5+2}}\right) = \log_2\left(x^2(3x+7)\right) - \log_2\left(\sqrt{x^5+2}\right)$$

$$= \log_2 x^2 + \log_2(3x+7) - \log_2\left((x^5+2)^{1/2}\right)$$

$$= 2\log_2 x + \log_2(3x+7) - \frac{1}{2}\log_2(x^5+2)$$

Now, $\frac{d}{dx}\left[\log_2\left(\frac{x^2(3x+7)}{\sqrt{x^5+2}}\right)\right]$

$$= \frac{d}{dx}\left[2\log_2 x + \log_2(3x+7) - \frac{1}{2}\log_2(x^5+2)\right]$$

$$= 2\frac{d}{dx}[\log_2 x] + \frac{d}{dx}[\log_2(3x+7)] - \frac{1}{2}\frac{d}{dx}[\log_2(x^5+2)]$$

$$= 2\cdot\frac{1}{x\ln 2} + \frac{1}{(3x+7)\cdot\ln 2}\cdot 3 - \frac{1}{2}\frac{1}{(x^5+2)\ln 2}\cdot 5x^4$$

$$= \frac{2}{x\cdot\ln 2} + \frac{3}{(3x+7)\ln 2} - \frac{5x^4}{2(x^5+2)\cdot\ln 2}$$

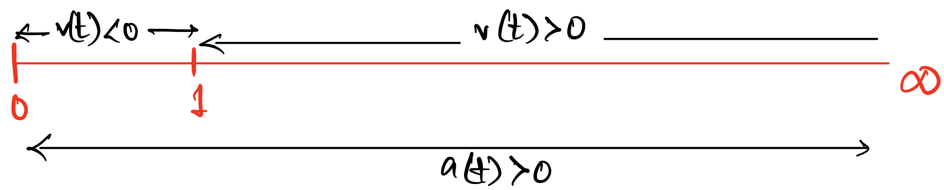
Ex. Suppose the position vector is given $s(t) = 4t^2 - 8t + 30$, $t \geq 0$.

Find the intervals where the particle is speeding up & slowing down.

$$s(t) = 4t^2 - 8t + 30$$

$$v(t) = s'(t) = 8t - 8 \begin{cases} \text{for } t > 1, & v(t) > 0 \\ \text{for } t < 1, & v(t) < 0 \end{cases}$$

$$a(t) = s''(t) = 8 \quad , \text{for all } t \geq 0$$



So, for $0 < t < 1$, $v(t) < 0$ & $a(t) > 0$

$\Rightarrow$ the particle is slowing down on $(0, 1)$

For $t > 1$, $v(t) > 0$ & $a(t) > 0$

$\Rightarrow$ the particle is speeding up on $(1, \infty)$